WADD TECHNICAL NOTE 60-201

TENSION, TORSION AND COLUMN PROPERTIES OF COMMERCIALLY PURE TITANIUM TUBING AT ROOM TEMPERATURE

Capt. Robert G. Henning

Materials Central

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Project No. 7351

WRIGHT AIR DEVELOPMENT DIVISION
AIR RESEARCH AND DEVELOPMENT COMMAND
UNITED STATES AIR FORCE
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

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FOREWORD

This report was prepared by the Strength and Dynamics Branch, Metals and Ceramics Division. The work was initiated under Project No. 7351, "Metallic Materials", Task No. 73521, "Behavior of Metals". It was administered under direction of the Materials Central, Directorate of Advanced Systems Technology, Wright Air Development Division, with Capt. Robert G. Henning acting as project engineer.

This report covers work conducted from September 1958 to March 1960.

ABSTRACT

Tensile, torsion and column properties were determined at room temperature on one heat of commercially pure titanium A 55 tubing in 1-inch and 3/8-inch outside diameters and varying wall thicknesses.

Typical stress-strain diagrams are presented for torsion and tension. Torsion/tension and column/tension ratios vs diameter/wall thickness graphs are presented.

PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDER:

W.J. Trapp

W.J. TRAPP
Chief, Strength and Dynamics Branch
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MATERIALS

The materials tested were type A 55 commercially pure titanium tubing purchased from Superior Tube Company of two outside diameters, one-inch and 3/8-inch and twelve different wall thicknesses. The 3/8-inch diameter tubes had nominal wall thicknesses of 0.006, 0.009, 0.015, 0.020, 0.030, 0.035 and 0.045 inches. The one-inch diameter tubes had nominal wall thicknesses of 0.025, 0.033, 0.040, 0.050 and 0.065 inches. All tubing was processed from one heat. Chemical analysis of the tubing presented in Table 1, is typical of the commercially pure titanium, A 55.

PROCEDURES AND TEST EQUIPMENT

The cross sectional areas were calculated from tubing length, weight and density measurements. The outside diameter of each tube was determined from several micrometer measurements.

Tensile testing for the tubes was performed on a 50,000 lb. capacity Baldwin FGT universal testing machine. Strains were measured autographically with an O. S. Peters SR-4 type extensometer and recorder, except for five one-inch-length specimens (one from each wall thickness), Four were measured with Tuckerman optical extensometers and one with two type A-1 SR-4 strain gages, and two Baldwin SR-4 strain meters.

Column testing for all tubes was performed on a 20,000 lb. capacity Tinius-Olsen universal testing machine using a flat compression plate on the stationary head and a spherical seat compression plate on the moving head. Careful alignment of the specimens was made to assure axial loading.

All torsion specimens were tested on a modified 3,000 inch-pound capacity Tinius-Olsen torsion testing machine. Torque was measured from a calibrated SR-4 load cell cantilever beam, and the twist by a mechanical troptometer.

All tension, torsion, and column specimens were tested by using a strain rate of 0.005 inches/inch/minute to the yield stress, and a crosshead travel rate of 0.05 inches/minute to failure. An exception was made for the one-inch diameter tube tensile tests which were made on the Tuckerman Optical Extensometer; the load was intermittently applied for strain reading purposes. The tension, torsion, and column specimens are shown in Figure 1.

RESULTS AND DISCUSSION

Three specimens were used in each type test and each diameter to thickness (D/t) ratio was tested at room temperature. Results for all tension tests are shown in Table 2. All tensile stress-strain curves were of the same nature; a typically shaped curve is presented in Figure 2. Tensile properties as a function of D/t ratio are presented in

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Figures 3 and 4. Tensile and ultimate strengths did not vary significantly with D/t ratio in the 3/8-inch tubing but the elongation decreased linearly when D/t ratio increased.

Tensile data vs D/t ratio for the one-inch tubing were widely scattered. Since the tubing was all processed from one heat of material, processing variables such as strain hardening or insufficient annealing could be responsible for the scatter.

The torsion specimens were tested by using a twelve-inch free twist length for the one-inch diameter tubes, and a four-and ten-inch free twist length for the 3/8-inch diameter tubes.

Results of torsion tests on the one-inch diameter tubing are presented in Table 3, and on the 3/8-inch tubing in Table 4. A typically shaped stress-strain curve in torsion is presented in Figure 5. Figures 6, 7 and 8 indicate the relationship of torsional properties, as a ratio of tensile properties, to D/t ratio. All yield ratios showed a slight linear increase with decreasing D/t ratios. One odd point on the curve for the one-inch tubes had a much lower yield ratio at a D/t value of 16 than expected, however, no explanation has been found for this behavior. The modulus of rupture/ultimate tensile strength ratio vs D/t ratio curves showed an increasing slope as the D/t ratio decreased. Figure 9 is a composite graph of the torsional to tensile properties ratio and shows that the slope of the yield ratio curves were approximately equal for the 3/8-inch and one-inch tubes but each at different strength levels. The rate of increase of this modulus of rupture to ultimate ratio curves was greater for the 3/8-inch tubes than for the one-inch tubes.

Results of the column tests are tabulated in Table 5. A graph which shows plots of Buckling stress/tensile yield ratio vs slenderness ratio, L'/ρ, is presented in Figure 10. Each point plotted for the 3/8-inch tubing is an average of seven values, one for each wall thickness. The points for the one-inch tubing are an average of 5 values for $L'/\rho = 20$, 4 for $L'/\rho = 40$ and 60, 3 for $L'/\rho = 80$ and 2 for $L'/\rho = 100$, 120 and 140. This was necessary because of the limited supply of material available, the length of tubing required for $L'/\rho = 140$ was 48 inches for the 0.065-inch wall material. Figure 10 reveals very good correlation between column properties of the one-inch and the 3/8-inch tubing.

All tensile, torsion,and column failures were of a ductile nature. The tensile specimens were the only tests that failed with an actual rupture of the material. Torsion specimens failed with a tube collapse and column specimens failed by tube collapse or by tube bending.

CONCLUSIONS

The chemical composition of these tubes is typical of A 55 titanium alloy.

The tensile properties of the 3/8-inch diameter tubing were consistent throughout the D/t range with the exception of the elongation which decreased as the D/t ratio increased. The tensile properties of the one-inch diameter tubing were very inconsistent, suggesting a nonconsistent anneal or excessive strain hardening in various tubes. The one-inch tubing properties, however, provided a good basis for correlation with other properties such as torsion,and column properties. It would therefore, be possible reasonably to predict torsion or column stresses from tensile data.

Torsional yield/tensile yield ratios decreased with increasing D/t ratios for all the tubes. The 3/8-inch tubes with a four-inch free twist length showed a 4 per cent higher yield strength ratio than the 3/8-inch tubes with a 10-inch free twist length based on the average curves.

The modulus of rupture/ultimate tensile strength ratio vs D/t ratio curve increased with decreasing D/t ratios with a greater rate of increase for the 3/8-inch tubing than the one-inch tubing. The 3/8-inch tubing curves were essentially the same for the 4-inch and 10-inch free twist length specimens.

Column buckling strength/tensile yield strength ratio vs slenderness ratio, L'/ρ, curves were equal for both one-inch and 3/8-inch tubing.

Table 1

Chemical Composition of A 55 Titanium Sheet

Diameter	Wall Thickness	% Carbon	% Iron	% Oxygen	Hydrogen ppm	Nitrogen ppm
1.000	0.025	0.07	0.21	0.127	43	115
1.000	0.033	0.01	0.11	0.095	46	60
1.000	0.040	0.07	0.22	0.131	55	150
1.000	0.050	0.11	0.40	0.083	43	140
1.000	0.065	0.02	0.10	0.102	35	130
0.375	0.006	0.04	0.13	0.088	29	60
0.375	0.009	0.03	0.10	0.115	44	60
0.375	0.014	0.02	0.12	0.092	30	70
0.375	0.019	0.01	0.11	0.105	40	70
0.375	0.030	0.01	0.13	0.105	47	55
0.375	0.035	0.02	0.11	0.099	52	70
0.375	0.045	0.01	0.12	0.090	41	130

Table 2

Tensile Results on Titanium Tubing of One-inch and 3/8-inch O. D.

Specimen Number	Nominal Wall Thickness	(D/t)	Percent Elongation in 2 inches	Yield Strength, psi (0.2% Offset)	Ultimate Tensile Strength, psi	Modulus of Elasticity psi x 10^6
* T-3	0.0240	42	33.0	52,300	76,900	14.3
T-4	0.0240	42	17.0	- (3)	75,400	15.5 (1)
T-6	0.0240	42	11.0	52,900	73,000	13.5
Avg	-	-	20.3	52,600	75,100	14.4
T-7	0.0325	32	53.0	38,300	58,800	14.1
T-8	0.0325	32	50.0	38,700	59,800	12.6
T-9	0.0325	32	50.0	37,100	59,700	15.1 (2)
Avg	-	-	50.3	38,000	59,400	13.9
T-1	0.0385	27	14.0	48,600	75,200	14.2
T-2	0.0385	27	19.0	- (3)	75,100	15.0 (1)
T-5	0.0385	27	33.0	51,600	74,400	13.6
Avg	-	-	22.3	50,100	74,900	14.3
T-10	0.0480	21	38.0	45,800	70,800	15.2
T-11	0.0480	21	31.0	41,100	67,500	15.2
T-12	0.0480	21	40.0	42,200	69,000	16.1 (1)
Avg	-	-	36.3	43,000	69,100	15.5
T-13	0.0650	16	42.0	48,700	66,600	15.2
T-14	0.0650	16	42.0	47,900	66,800	15.0
T-15	0.0650	16	41.0	- (3)	66,100	16.1 (1)
Avg	-	-	41.7	48,300	66,500	15.4

* Above data is for 1-inch tubing

Notes

(1) Tuckerman Extensometer Used
(2) SR-4 Strain Gages Used
(3) Value not obtained

Table 2 (Cont'd)

Specimen Number	Nominal Wall Thickness	(D/t)	Percent Elongation in 1 1/2 inches	Yield Strength, psi (0.002 $\frac{inch}{inch}$ Offset)	Ultimate Tensile Strength, psi	Modulus of Elasticity psi x 10^6
*						
T-1	0.0055	68	- (3)	38,800	60,000	14.6
T-2	0.0055	68	- (3)	38,400	60,000	14.3
T-3	0.0055	68	24.0	38,600	60,300	14.9
Avg	-	-	24.0	38,600	60,100	14.6
T-4	0.0085	44	- (3)	46,800	67,400	15.1
T-5	0.0085	44	- (3)	46,800	67,200	15.1
T-6	0.0085	44	27.0	47,200	67,000	15.6
Avg	-	-	27.0	46,900	67,200	15.3
T-7	0.0130	28	32.0	42,400	62,800	14.9
T-8	0.0130	28	32.0	41,600	62,200	15.1
T-9	0.0130	28	32.0	42,200	63,600	15.4
Avg	-	-	32.0	42,100	62,900	15.1
T-10	0.0200	19	37.0	44,300	63,900	15.3
T-11	0.0200	19	37.0	43,400	63,300	14.6
T-12	0.0200	19	35.0	44,600	63,000	14.8
Avg	-	-	36.3	44,100	63,400	14.9
T-13	0.0285	13	37.0	43,500	63,500	15.1
T-14	0.0285	13	37.0	43,900	63,700	15.2
T-15	0.0285	13	39.0	43,500	63,500	15.2
Avg	-	-	37.7	43,600	63,600	15.2
T-16	0.0340	11	39.0	43,200	65,000	15.4
T-17	0.0340	11	41.0	43,600	64,900	15.4
T-18	0.0340	11	37.0	43,200	64,600	15.2
Avg	-	-	39.0	43,300	64,800	15.3
T-19	0.0415	9	40.0	44,600	65,800	15.7
T-20	0.0415	9	37.0	44,600	65,600	15.4
T-21	0.0415	9	37.0	44,600	65,600	15.3
Avg	-	-	38.0	44,600	65,700	15.5

* Above data is for 3/8-inch tubing

Note

(3) Value not obtained

Table 3

Torsion Results on Titanium Tubing of One-inch O.D. and 12-inch Free Twist Length

Specimen Number	Wall Thickness	D/t	Yield Strength psi	Torsional Yield / Tensile Yield	Modulus Of Rupture, psi	Modulus of Rupture / Tensile Ultimate	Torsional Modulus of Elasticity x 10^6 psi
A-1	0.0240	42	32,300	0.62	35,100	0.47	6.5
A-2	0.0240	42	36,500	0.69	39,200	0.52	6.7
A-3	0.0240	42	36,200	0.69	38,500	0.51	6.3
Avg	-	-	34,700	0.66	38,500	0.51	6.5
A-4	0.0325	32	26,800	0.70	33,900	0.57	6.4
A-5	0.0325	32	27,400	0.72	35,000	0.59	6.6
A-6	0.0325	32	27,200	0.72	34,100	0.57	6.4
Avg	-	-	27,100	0.71	34,300	0.58	6.5
A-7	0.0385	27	34,000	0.68	42,100	0.56	6.2
A-8	0.0385	27	34,300	0.68	41,800	0.56	6.4
A-9	0.0385	27	34,700	0.69	43,100	0.57	6.4
Avg	-	-	34,300	0.68	42,300	0.56	6.3
A-10	0.0480	21	30,500	0.71	41,600	0.60	6.8
A-11	0.0480	21	29,700	0.69	40,300	0.58	6.5
A-12	0.0480	21	30,100	0.70	43,200	0.62	7.0
Avg	-	-	30,100	0.70	41,700	0.60	6.8
A-13	0.0650	16	27,000	0.56	43,100	0.65	6.6
A-14	0.0650	16	32,100	0.66	45,200	0.68	6.4
A-15	0.0650	16	-	-	-	-	-
Avg	-	-	29,500	0.61	44,100	0.66	6.5

Table 4

Torsion Results on Titanium Tubing 3/8-inch O.D. and Four-inch and Ten-inch Free Twist Length

Specimen Number	Wall Thickness	D/t	Twist at Failure Degree/inch Length	Yield Strength psi	Torsional Yield / Tensile Yield	Modulus of Rupture psi	Modulus of Rupture / Tensile Ultimate	Torsional Modulus of Elasticity x 10^6 psi
A-1	0.0055	68	1	23,000	0.60	23,000	0.38	5.7
A-2	0.0055	68	1	21,500	0.56	21,500	0.36	5.7
A-3	0.0055	68	1	22,300	0.58	22,300	0.37	5.0
Avg	-	-	1	22,300	0.58	22,300	0.37	5.5
A-4	0.0085	44	-	25,400	0.54	27,500	0.41	5.4
A-5	0.0085	44	5	26,800	0.57	29,100	0.43	5.4
A-6	0.0085	44	5	25,200	0.54	27,900	0.41	5.6
Avg	-	-	5	25,800	0.55	28,200	0.42	5.5
A-7	0.0130	28	15	26,200	0.62	29,200	0.46	5.6
A-8	0.0130	28	15	26,100	0.62	34,300	0.54	5.9
A-9	0.0130	28	15	24,100	0.57	32,600	0.52	5.6
Avg	-	-	15	25,500	0.61	32,100	0.51	5.7
A-10	0.0200	19	-	24,200	0.55	37,400	0.59	6.0
A-11	0.0200	19	-	28,500	0.65	38,200	0.60	5.8
A-12	0.0200	19	45	24,800	0.56	39,200	0.62	5.7
Avg	-	-	45	25,800	0.58	38,300	0.60	5.8
A-13	0.0285	13	105	27,100	0.62	44,600	0.70	5.8
A-14	0.0285	13	90	26,500	0.61	44,000	0.69	5.8
A-15	0.0285	13	105	26,600	0.61	44,600	0.70	5.7
Avg	-	-	100	26,700	0.61	44,400	0.70	5.8

Table 4 (Cont'd)

Specimen Number	Wall Thickness	D/t	Twist at Failure Degree/inch Length	Yield Strength psi	Torsional Yield / Tensile Yield	Modulus of Rupture psi	Modulus of Rupture / Tensile Ultimate	Torsional Modulus (G) x 10^6 psi
** A-16	0.0340	11	-	26,800	0.62	35,500*	-	5.6
A-17	0.0340	11	177	25,400	0.59	50,800	0.79	5.7
A-18	0.0340	11	153	27,400	0.63	49,300	0.76	5.7
Avg	-	-	165	26,500	0.61	50,000	0.77	5.7
A-19	0.0415	9	255	27,700	0.62	53,100	0.98	5.7
A-20	0.0415	9	-	27,200	0.61	56,600	0.86	6.2
A-21	0.0415	9	-	26,800	0.60	59,100	0.90	5.7
Avg	-	-	255	27,200	0.61	57,900	0.88	5.9

* - Not Included in Average

** "A" specimen numbers are 4-inch free twist length.

Table 4 (Cont'd)

Specimen Number	Wall Thickness	D/t	Twist at Failure Degree/inch Length	Yield Strength psi	Torsional Yield / Tensile Yield	Modulus of Rupture psi	Modulus of Rupture / Tensile Ultimate	Torsional Modulus (G) x 10^6 psi
* B-1	0.0055	68	-	20,800	0.54	20,800	0.35	5.7
B-2	0.0055	68	-	20,700	0.54	20,700	0.34	5.3
B-3	0.0055	68	1/2	21,400	0.55	21,400	0.36	5.5
Avg	-	-	1/2	21,000	0.54	21,000	0.35	5.5
B-4	0.0085	44	3	25,600	0.55	29,800	0.44	5.7
B-5	0.0085	44	3	25,300	0.54	29,300	0.44	5.8
B-6	0.0085	44	3	25,900	0.55	29,300	0.44	5.6
Avg	-	-	3	25,600	0.55	29,500	0.44	5.7
B-7	0.0135	28	8	26,100	0.62	33,400	0.53	5.5
B-8	0.0135	28	8	25,900	0.61	33,200	0.53	5.7
B-9	0.0135	28	8	26,400	0.63	33,900	0.54	5.8
Avg	-	-	8	26,100	0.62	33,500	0.53	5.7
B-10	0.0205	19	55	24,000	0.54	37,500	0.59	5.7
B-11	0.0205	19	52	23,500	0.53	37,100	0.58	5.6
B-12	0.0205	19	50	23,800	0.54	37,400	0.59	5.5
Avg	-	-	52	23,800	0.54	37,300	0.59	5.6
B-13	0.0280	13	93	26,100	0.60	43,000	0.68	5.7
B-14	0.0280	13	93	26,000	0.60	41,900	0.66	5.5
B-15	0.0280	13	93	26,500	0.61	43,000	0.67	5.3
Avg	-	-	93	26,200	0.60	42,600	0.67	5.5
B-16	0.0335	11	180	25,100	0.58	51,800	0.80	5.7
B-17	0.0335	11	170	24,600	0.57	46,900	0.72	5.7
B-18	0.0335	11	180	24,400	0.56	50,600	0.78	5.6
Avg	-	-	177	24,700	0.57	49,800	0.77	5.7
B-19	0.0420	9	-	26,600	0.60	56,400	0.86	5.7
B-20	0.0420	9	240	26,600	0.60	56,200	0.85	5.6
B-21	0.0420	9	246	26,500	0.59	56,900	0.87	5.7
Avg	-	-	243	26,600	0.60	56,500	0.86	5.7

* "B" specimen numbers are 10-inch free twist length.

Specimen Number	Slenderness Ratio L'/ρ	Buckling Stress	Buckling Stress / Average Yield Stress
** C-3	20	60,100	1.14
C-4	20	50,500	1.33
C-9	20	66,500	1.33
C-13	20	60,000	1.39
C-18	20	63,600	1.32
Average	20	62,100	1.30
C-2	40	56,000	1.06
C-5	40	43,300	1.14
C-10	40	56,900	1.13
C-14	40	49,400	1.15
Average	40	52,100	1.12
C-1	60	51,400	0.98
C-6	60	38,400	1.01
C-15	60	44,200	1.02
C-20	60	47,600	0.99
Average	60	45,400	1.00
C-7	80	33,600	0.88
C-16	80	38,000	0.88
Average	80	35,800	0.88
C-17	100	33,100	0.77
C-19	100	35,500	0.73
Average	100	34,300	0.75
C-11	120	35,700	0.71
C-21	120	30,800	0.64
Average	120	33,300	0.67
C-8	140	20,900	0.55
C-12	140	27,900	0.56
Average	140	24,400	0.55
*** CA-1	20	45,800	1.19
CB-1	20	32,200*	0.69*
CC-1	20	57,400	1.36
CD-1	20	58,200	1.32
CE-1	20	59,700	1.37
CF-1	20	61,800	1.43
CG-1	20	65,800	1.47
Average	20	58,100	1.36

Table 5

Column Results on Titanium Tubing One-inch and 3/8-inch O.D.

* Not included in average.

** All single letter specimen numbers are 1-inch tubing.

*** All double letter specimen numbers are 3/8-inch tubing.

Table 5 (Cont'd)			
Specimen Number	Slenderness Ratio L'/ρ	Buckling Stress	Buckling Stress / Average Yield Stress
CA-2	40	40,700	1.05
CB-2	40	48,500	1.03
CC-2	40	50,500	1.20
CD-2	40	45,300	1.03
CE-2	40	50,100	1.15
CF-2	40	51,000	1.18
CG-2	40	47,800	1.07
Average	40	47,700	1.10
CB-3	60	42,100	0.90
CC-3	60	46,000	1.09
CD-3	60	41,500	0.94
CE-3	60	44,700	1.02
CF-3	60	40,500	0.94
CG-3	60	42,600	0.95
Average	60	42,900	0.97
CA-3	80	27,800	0.72
CB-4	80	38,100	0.81
CC-4	80	37,900	0.90
CD-4	80	37,300	0.85
CE-4	80	39,700	0.91
CF-4	80	38,000	0.88
CG-4	80	37,100	0.83
Average	80	36,500	0.84
CA-4	100	30,100	0.78
CB-5	100	34,100	0.73
CC-5	100	35,300	0.84
CD-5	100	33,100	0.75
CE-5	100	34,000	0.78
CF-5	100	33,500	0.77
CG-5	100	32,400	0.73
Average	100	33,200	0.77
CA-5	120	25,600	0.66
CB-6	120	28,100	0.60
CC-6	120	30,700	0.73
CD-6	120	28,900	0.66
CE-6	120	29,600	0.68
CF-6	120	30,000	0.69
CG-6	120	27,800	0.62
Average	120	28,700	0.66
CA-6	140	22,400	0.58
CB-7	140	22,400	0.48
CC-7	140	23,000	0.55

Table 5 (Cont'd)

Specimen Number	Slenderness Ratio L'/ρ	Buckling Stress	Buckling Stress / Average Yield Stress
CD-7	140	23,000	0.52
CE-7	140	26,100	0.60
CF-7	140	23,500	0.54
CG-7	140	24,100	0.54
Average	140	23,500	0.54

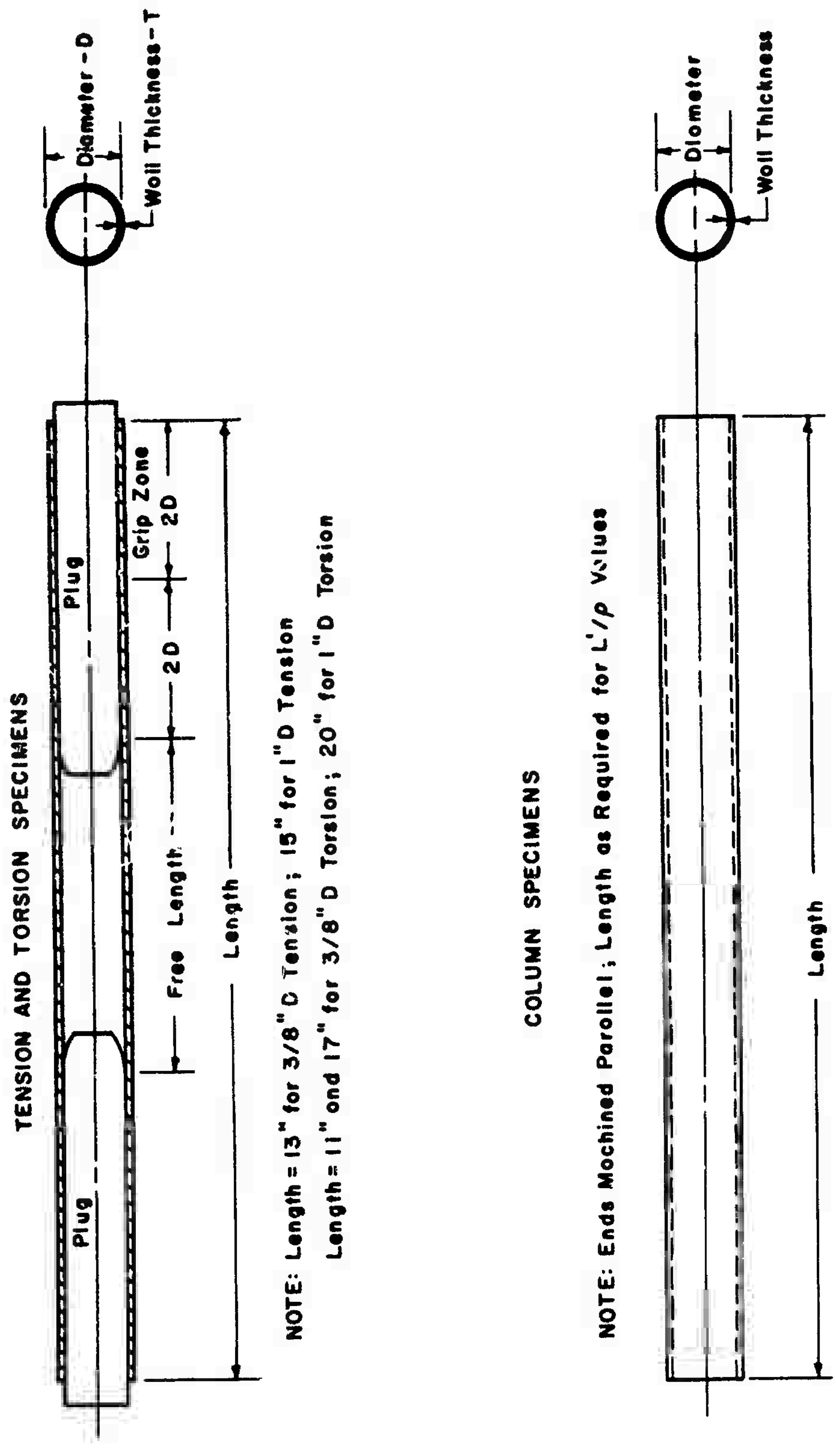

Figure 1. Tension, Torsion, and Column Specimens

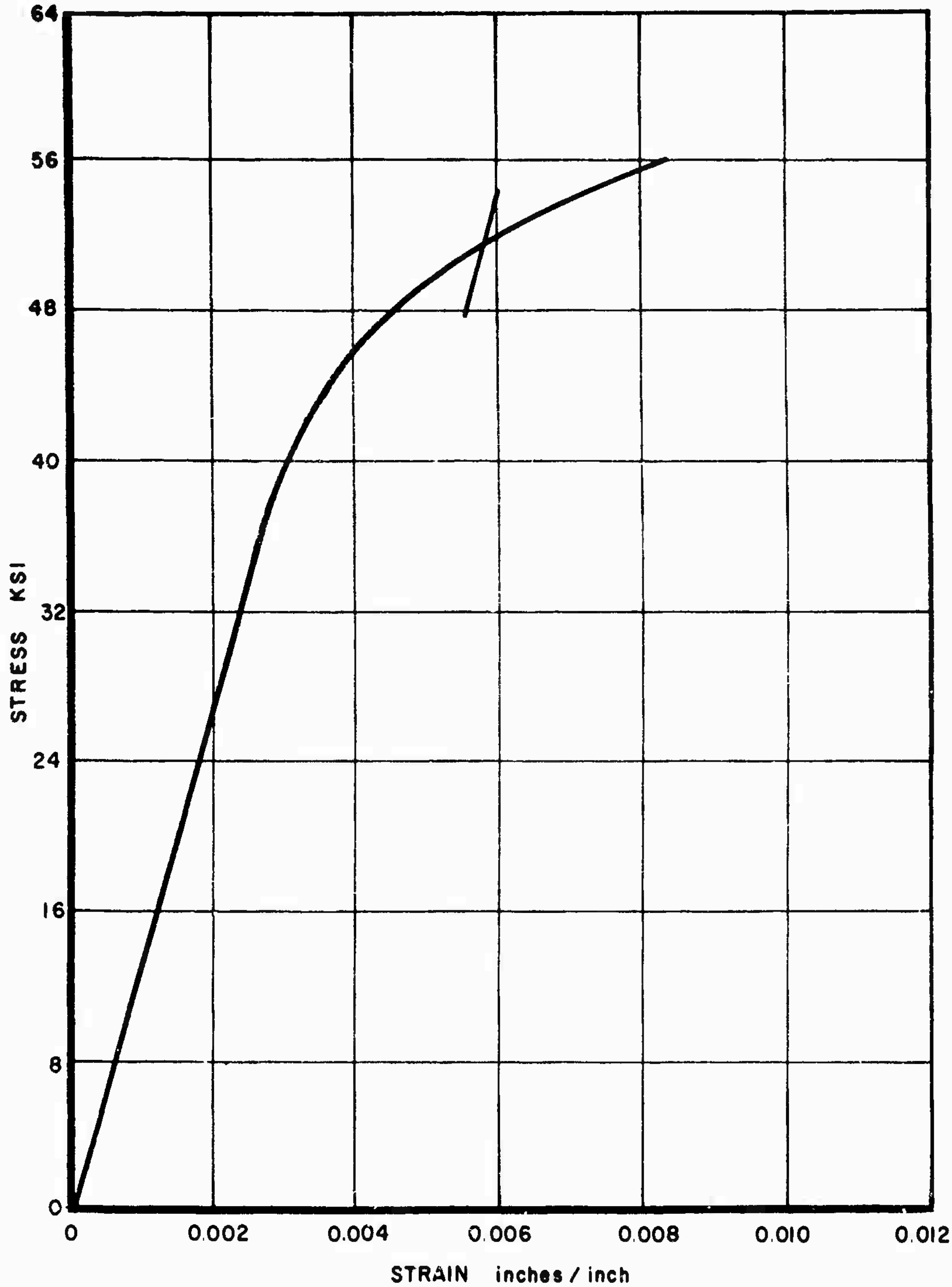

Figure 2. Typical Stress-Strain Curve for one-inch and 3/8-inch Titanium Tubing in Tension

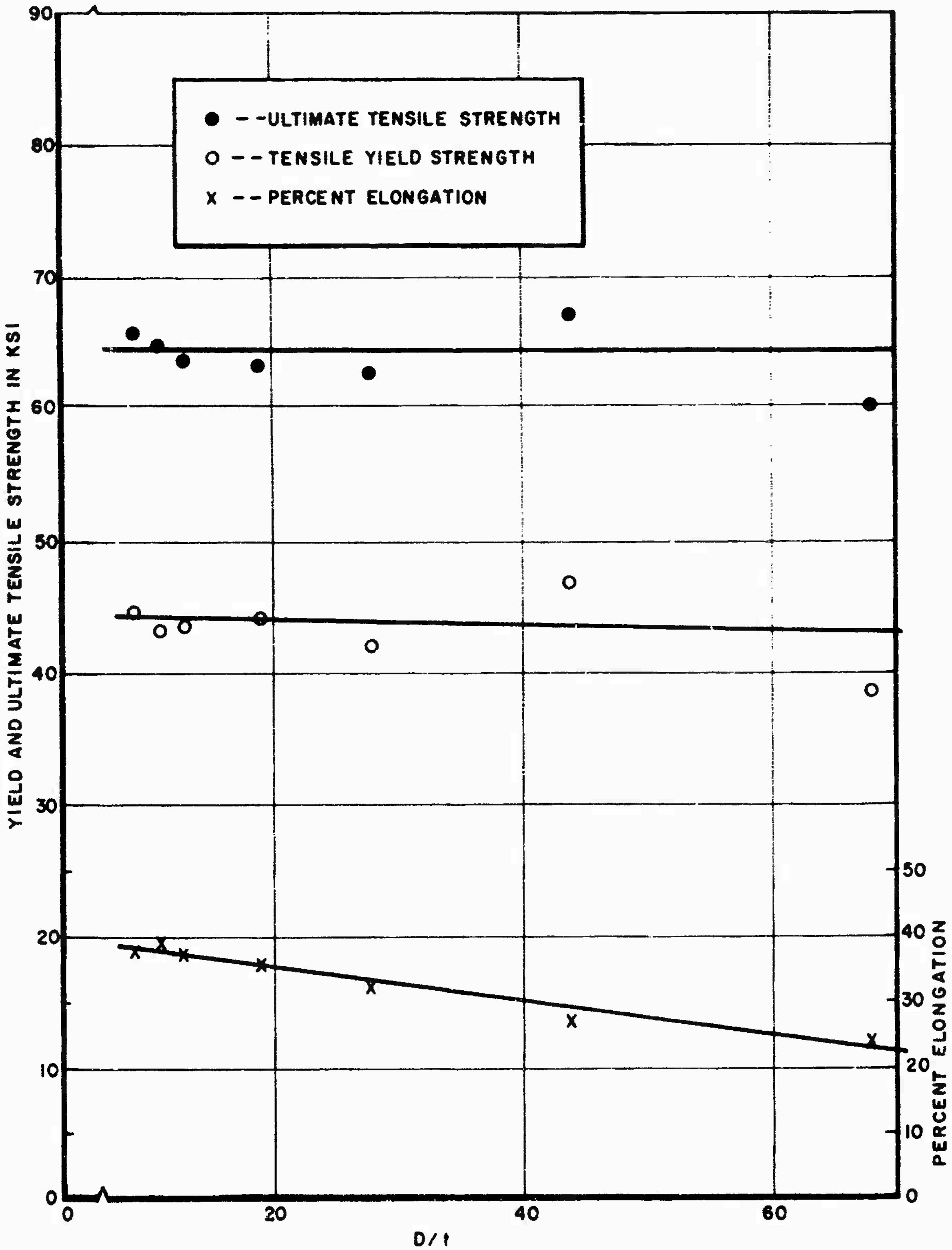

Figure 3. Tensile Properties vs D/t for 3/8-inch Titanium Tubing

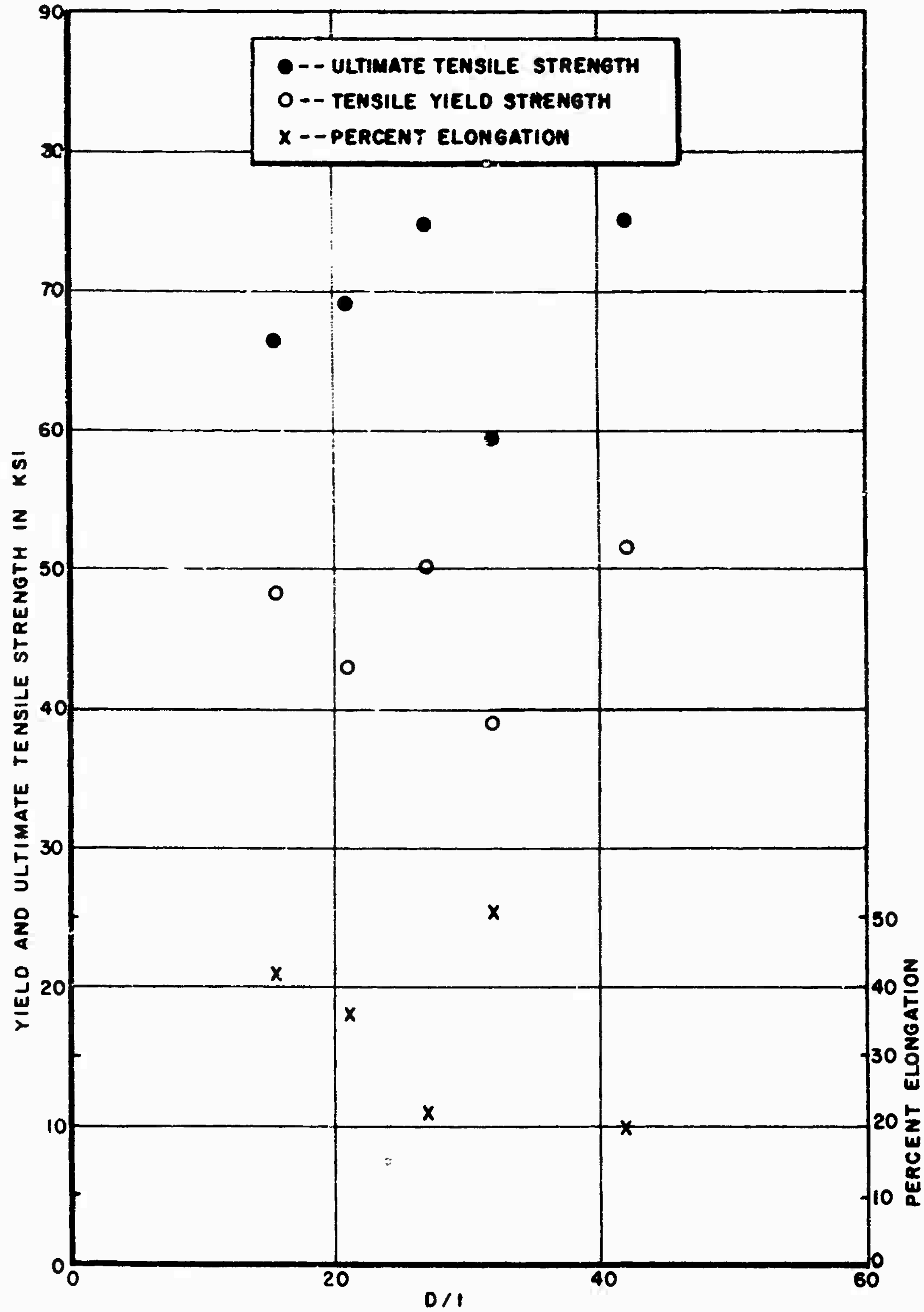

Figure 4. Tensile Properties vs D/t for one-inch Titanium Tubing

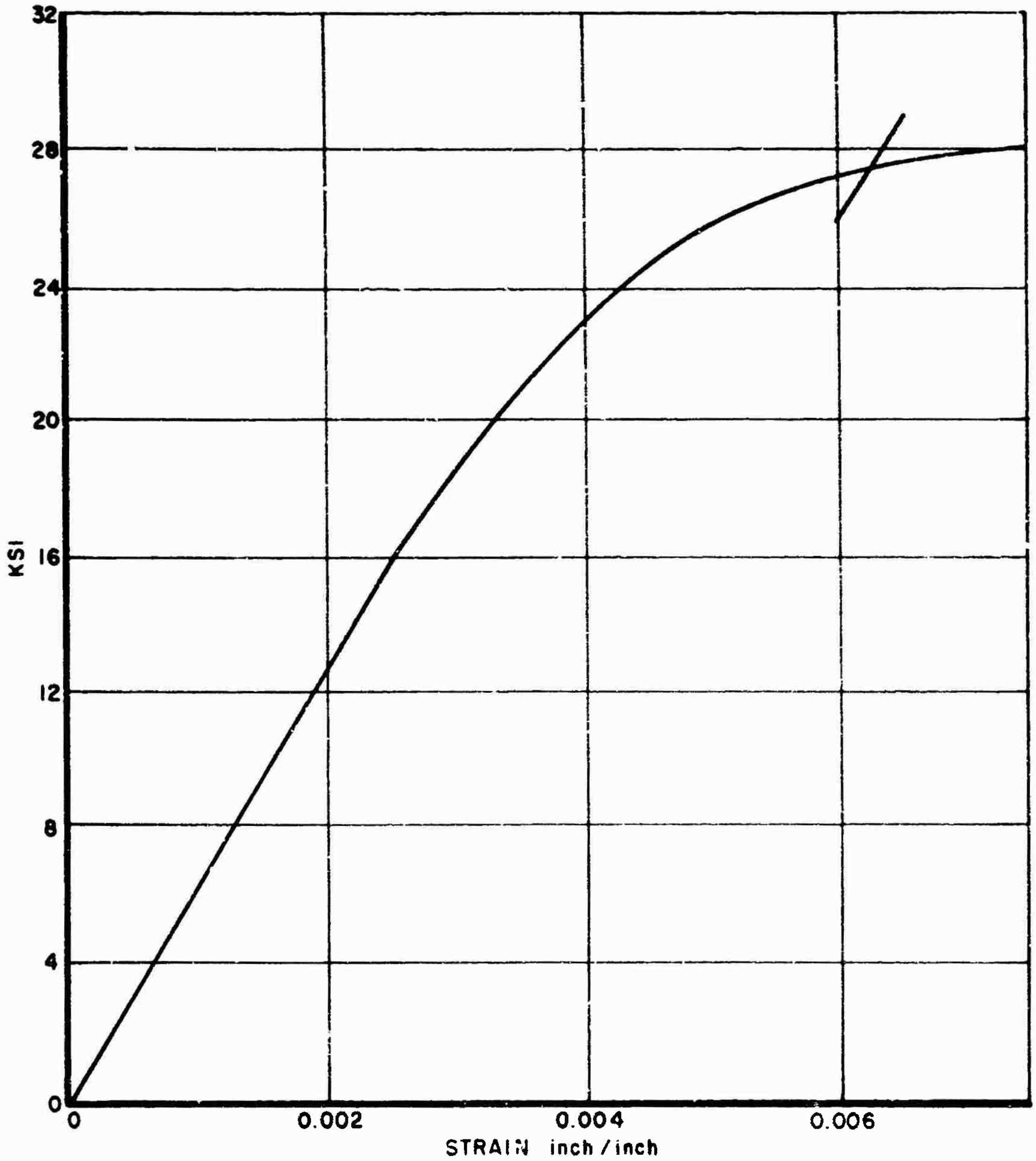

Figure 5. Typical Stress-Strain Curve in Torsion for one-inch and 3/8-inch Titanium Tubing

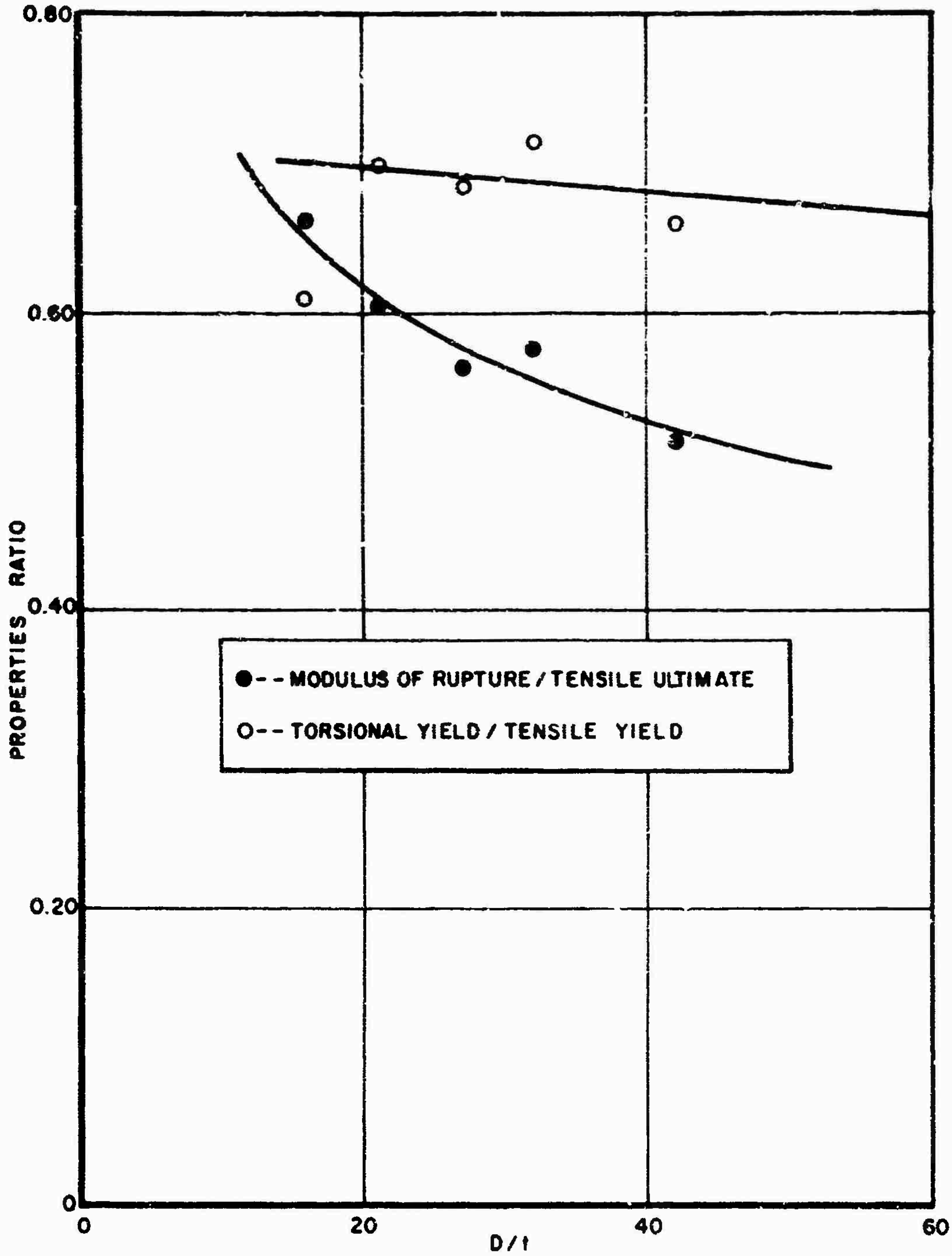

Figure 6. Properties Ratio vs D/t for one-inch O.D. Titanium Tubing, 12-inch Free Twist Length

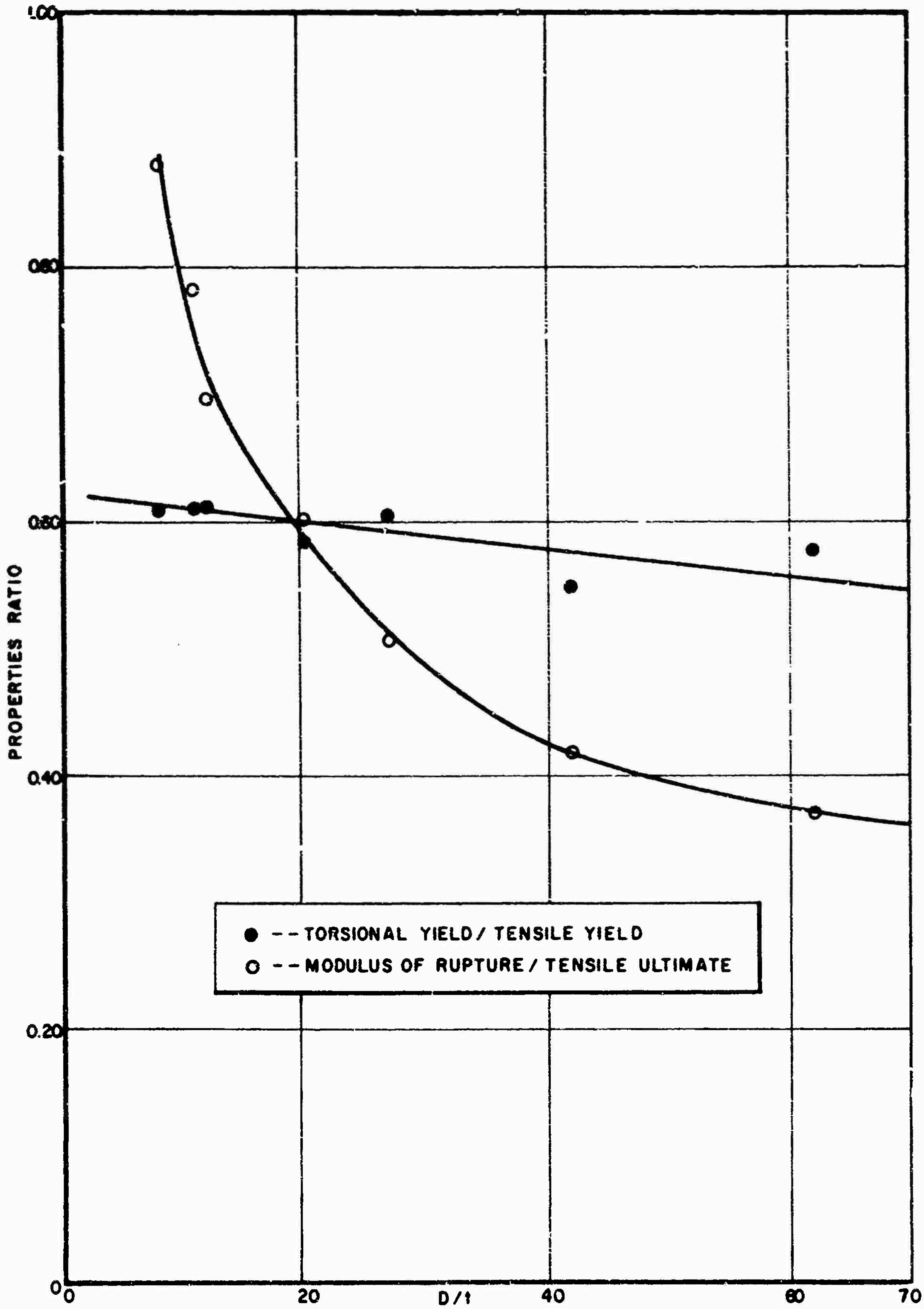

Figure 7. Properties Ratio vs D/t for 3/8-inch O.D. Titanium Tubing, 4-inch Free Twist Length

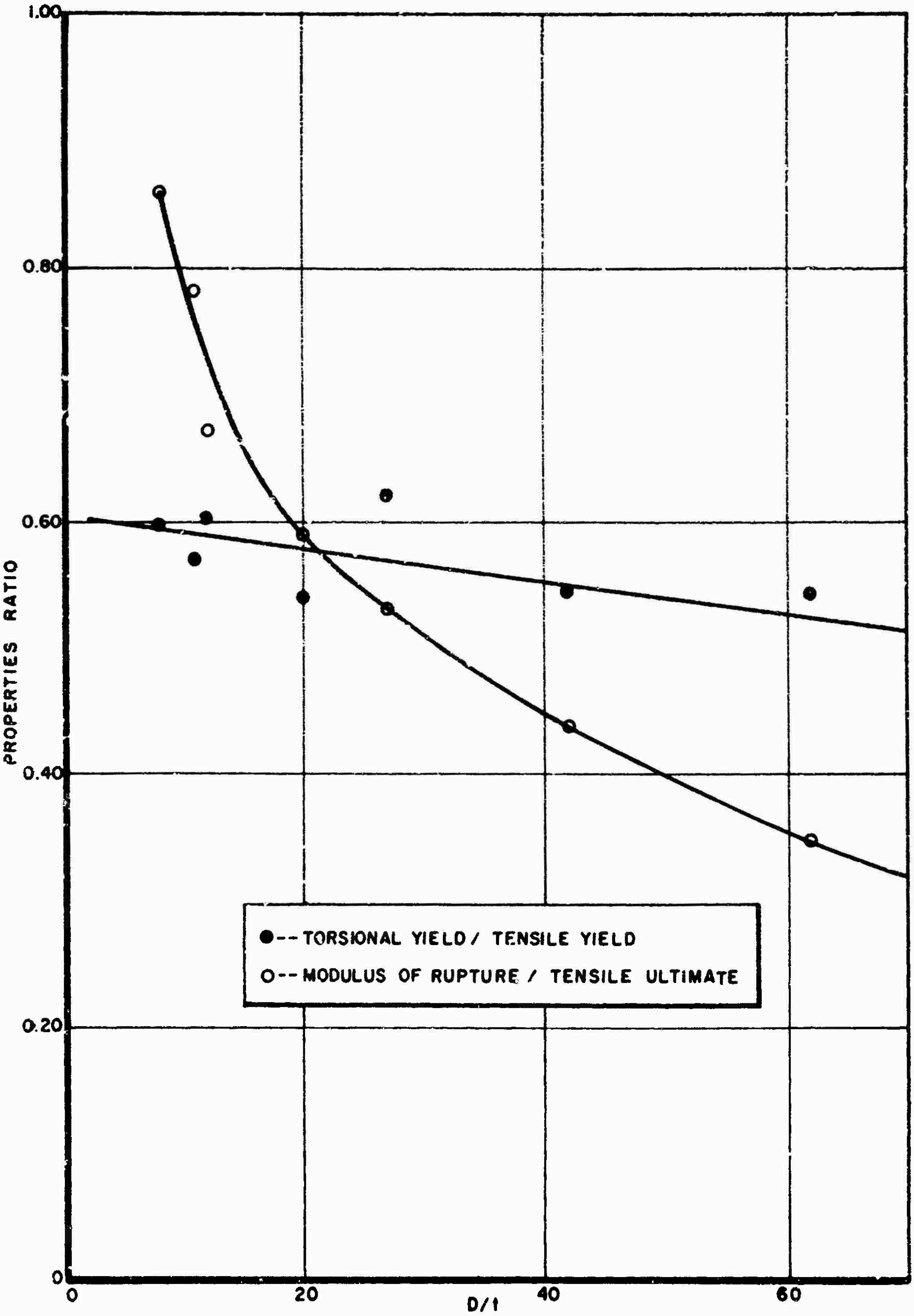

Figure 8. Properties Ratio vs D/t for 3/8-inch O.D. Titanium Tubing, 10-inch Free Twist Length

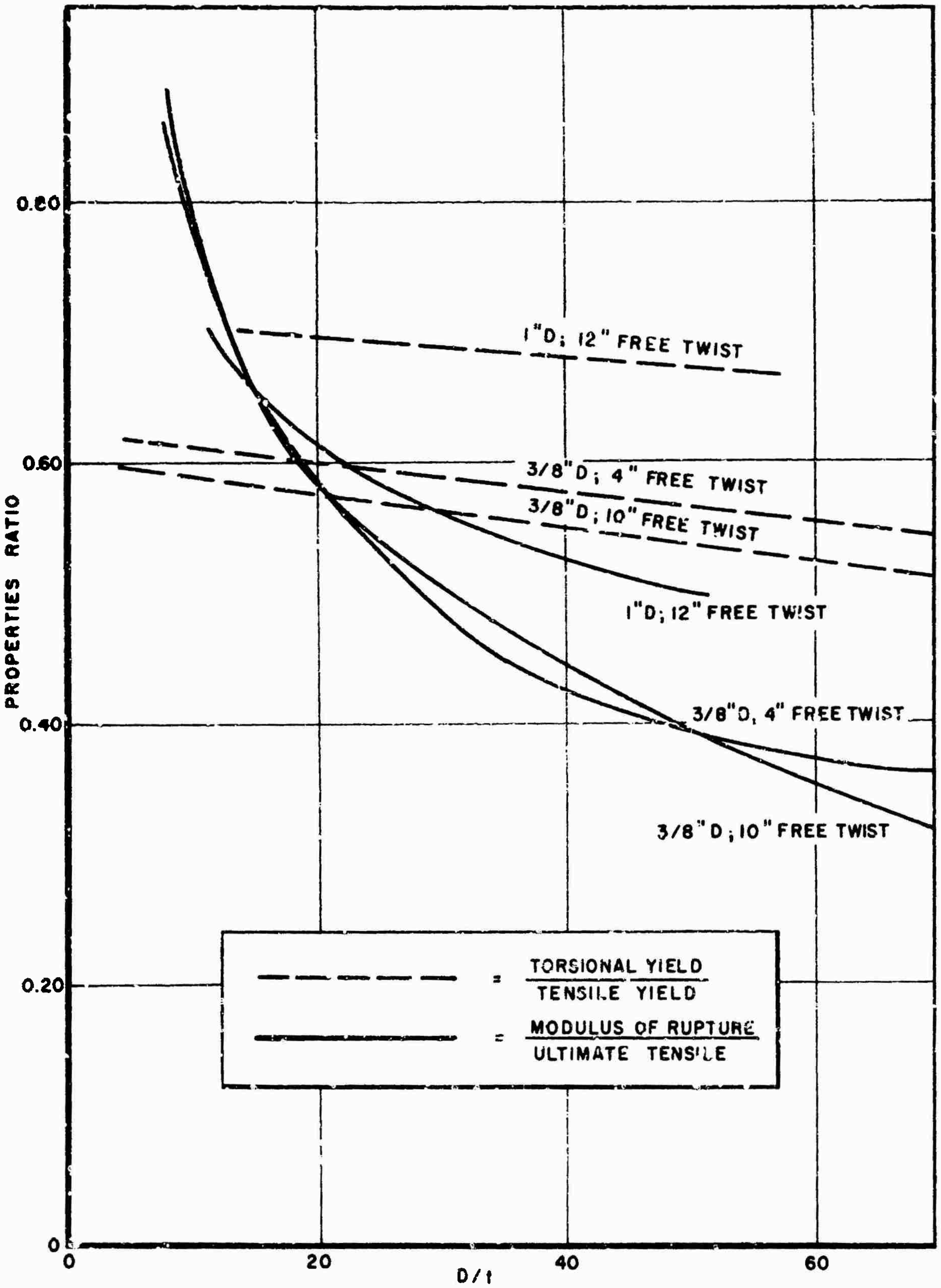

Figure 9. Properties Ratio vs D/t

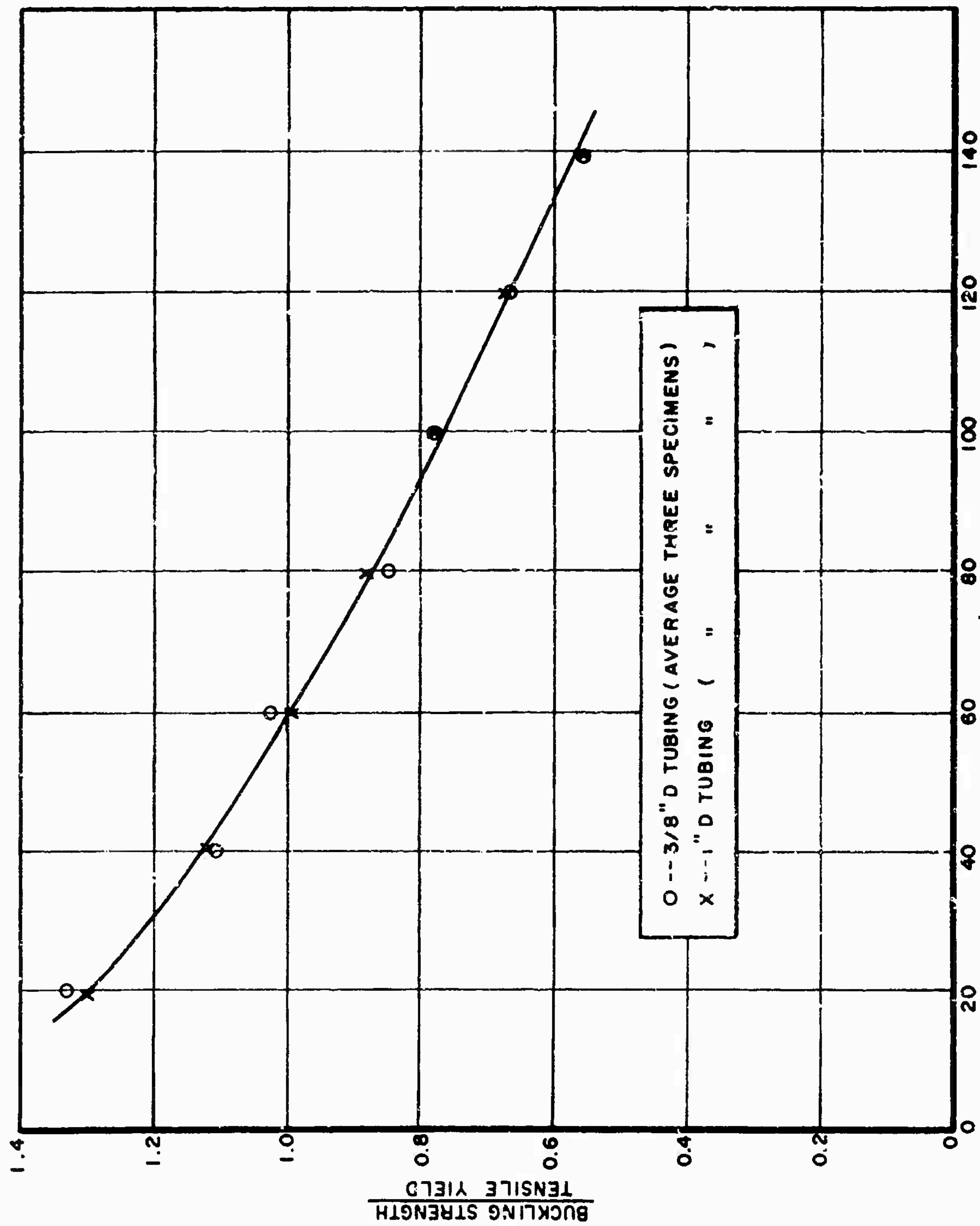

Figure 10. Titanium Tubing Column Tests Buckling Stress/Tensile Yield vs Slenderness Ratio